

Asymmetries for multivariate returns: an empirical approach

Asimmetrie nei rendimenti multivariati: un approccio empirico

Giovanni De Luca¹

Dipartimento di Statistica e Matematica per la Ricerca Economica, Università degli Studi di Napoli *Parthenope*, email: giovanni.deluca@uniparthenope.it

Nicola Loperfido

Istituto di Scienze Economiche, Università degli Studi di Urbino *Carlo Bo*, email: nicola.loperfido@uniurb.it

Riassunto: Nei modelli multivariati per l'eteroschedasticità, l'ipotesi di distribuzione normale è spesso lontana dall'evidenza empirica. In questo lavoro si intende evidenziare che l'analisi del momento terzo del vettore dei residui può essere di ausilio per individuare una migliore rappresentazione statistica dei rendimenti multivariati.

Keywords: Asymmetry, Multivariate GARCH, Skew-elliptical distributions.

1. Introduction

In the multivariate analysis of financial returns, academics and practitioners tend to focus on modelling conditional heteroskedasticity, without emphasis on the univariate asymmetry of each series and on other relationships. The traditional diagnostics based on mean and variance of the residuals should be enlarged to the analysis of higher moments.

2. The empirical analysis

Our work has an empirical genesis. We have analyzed the time series of daily percentual log-returns of DAX (Germany), CAC (France) and UKX (UK) indexes in the period 17-01-1995 - 02-05-2003 applying a multivariate Gaussian GARCH model with constant conditional correlation (Bollerslev, 1990). The analysis of the standardized residuals $\mathcal{E} = [\varepsilon_1 \ \varepsilon_2 \ \varepsilon_3]'$ can reveal possible inadequacies of the model. Going beyond the traditional diagnostics tools, we want to focus on the analysis of $E(\mathcal{E} \otimes \mathcal{E}' \otimes \mathcal{E})$, estimating

¹Address of correspondence: via Medina, 40, 80133 Napoli.

$$\hat{E}(\mathcal{E} \otimes \mathcal{E}' \otimes \mathcal{E}) = \hat{E} \begin{bmatrix} \varepsilon_1^3 & \varepsilon_1^2 \varepsilon_2 & \varepsilon_1^2 \varepsilon_3 \\ \varepsilon_1^2 \varepsilon_2 & \varepsilon_1 \varepsilon_2^2 & \varepsilon_1 \varepsilon_2 \varepsilon_3 \\ \varepsilon_1^2 \varepsilon_3 & \varepsilon_1 \varepsilon_2 \varepsilon_3 & \varepsilon_1 \varepsilon_3^2 \\ \varepsilon_1^2 \varepsilon_2 & \varepsilon_1 \varepsilon_2^2 & \varepsilon_1 \varepsilon_2 \varepsilon_3 \\ \varepsilon_1 \varepsilon_2^2 & \varepsilon_2^3 & \varepsilon_2^2 \varepsilon_3 \\ \varepsilon_1 \varepsilon_2 \varepsilon_3 & \varepsilon_2^2 \varepsilon_3 & \varepsilon_2 \varepsilon_3^2 \\ \varepsilon_1^2 \varepsilon_3 & \varepsilon_1 \varepsilon_2 \varepsilon_3 & \varepsilon_1 \varepsilon_3^2 \\ \varepsilon_1 \varepsilon_2 \varepsilon_3 & \varepsilon_2^2 \varepsilon_3 & \varepsilon_2 \varepsilon_3^2 \\ \varepsilon_1 \varepsilon_3^2 & \varepsilon_2 \varepsilon_3^2 & \varepsilon_3^3 \end{bmatrix} = \begin{bmatrix} -0.3209 & -0.2703 & -0.2824 \\ -0.2703 & -0.2153 & -0.2371 \\ -0.2824 & -0.2371 & -0.2480 \\ -0.2703 & -0.2153 & -0.2371 \\ -0.2153 & -0.1403 & -0.2092 \\ -0.2371 & -0.2092 & -0.2263 \\ -0.2824 & -0.2371 & -0.2480 \\ -0.2371 & -0.2092 & -0.2263 \\ -0.2480 & -0.2263 & -0.2473 \end{bmatrix}$$

Note that in the Gaussian case the theoretical matrix is a matrix of zeros.

Each element $\hat{E}(\varepsilon_i^3)$ with $i = 1, 2, 3$ in the matrix represents the distributional asymmetry. The three estimated values are negative implying that the skewness of the marginal distributions of the returns is not taken into account in the Gaussian multivariate GARCH model.

The elements $\hat{E}(\varepsilon_i^2 \varepsilon_j)$, with $i \neq j$, indicate what we intend to define as cross contemporaneous asymmetry. A high absolute return of index i is associated to a low return of index j , that is a high volatility of index i is associated to a low return of index j .

Finally the element $\hat{E}(\varepsilon_1 \varepsilon_2 \varepsilon_3)$ turns out to be negative. Actually, positive signs occur 50.12%, while negative signs occur 49.88%. However, the sum of the positive products is 765.36 while the sum of the negative products is -1387.05 . The latter sum mainly depends on the event *three negative residuals* which amounts at -1333.04 , (the corresponding sum for the the event *three positive residuals* is 714.62). A possible interpretation: when news are bad, returns, substantially moving together, tend to go down; when news are good, returns tend to go up but to a less extent. We will denote this feature as joint asymmetry.

In order to take into account these features, a multivariate model should allow asymmetry in the marginal distributions as well as the other asymmetries. In a multivariate GARCH model, the assumption of multivariate normality can be replaced by a more comprehensive multivariate skew-elliptical distribution (Branco and Dey, 2001), in particular the multivariate Skew-Normal distribution (Azzalini and Dalla Valle, 1996 and Genton, 2004) and the multivariate Skew- t distribution (Azzalini and Capitanio, 2004). On the other hand, the conditional volatility may be described by a formulation allowing for the leverage effect, such as the EGARCH or the TGARCH.

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